

Bellwork:

How many zeros are there in the polynomial:

$$f(x) = x^3 + 5x^2 + 4x + 20 = 0$$

Chapter 5.7: Apply the Fundamental Theorem of Algebra

The highest exponent in the polynomial tells you how many solutions/zeros/roots/factors there are.

Find all zeros: $-1, -1, 2, 2 \pm 3i$

$$f(x) = x^5 - 4x^4 + 4x^3 + 10x^2 - 13x - 14$$

$$\frac{p}{q} = \frac{14}{1} = \frac{1, 2, 7, 14}{1} = \boxed{\pm 1, \pm 2, \pm 7, \pm 14}$$

-1	1	-4	4	10	-13	-14
		-1	5	-9	-1	14
-1	1	-5	9	1	-14	0
		-1	6	-15	14	
2	1	-6	15	-14	0	
		2	-8	14		
			$1x^2 - 4x + 7$	0		

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 28}}{2}$$

$$x = \frac{4 \pm \sqrt{-12}}{2}$$

$$x = \frac{4 \pm 2\sqrt{3}i}{2}$$

$$x = 2 \pm \sqrt{3}i$$

$$2 + \sqrt{3}i, 2 - \sqrt{3}i$$

If $a+bi$ is a zero then $a-bi$ is also a zero

$$2 + 3i \quad 2 - 3i$$

If $a+\sqrt{b}i$ is a zero then $a-\sqrt{b}i$ is also a zero

$$2 + \sqrt{3}i \quad 2 - \sqrt{3}i$$

$$2 + \sqrt{5}i \quad 2 - \sqrt{5}i$$

~~2 + 3i~~

ex. write a polynomial of least degree with rational coefficients, a leading coefficient of 1 and 3 and $2+\sqrt{5}$ as zeros.

$$x = 3, 2+\sqrt{5}, 2-\sqrt{5}$$

$$(x-3)(x-(2+\sqrt{5}))(x-(2-\sqrt{5}))$$

$$(x-3)(x-2-\sqrt{5})(x-2+\sqrt{5})$$

$$(x-3)(x^2-4x-1)$$

$$x^3 - 4x^2 - 1x - 3x^2 + 12x + 3$$

$$x^3 - 7x^2 + 11x + 3$$

Sign Changes: (Descartes Rule)

+ real - the number of sign changes in $f(x)$ or less by even number(-2).

7, 5, 3, 1

- real - then number of sign changes in $f(-x)$ or less by even number(-2).

How many positive/negative real solutions?
Imaginary?

$$f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$$

+ real: 3, 1

$$f(-x) = (-x)^6 - 2(-x)^5 + 3(-x)^4 - 10(-x)^3 - 6(-x)^2 - 8(-x) - 8$$

$$f(-x) = x^6 + 2x^5 + 3x^4 + 10x^3 - 6x^2 + 8x - 8$$

- real: 3, 1

+ real	- real	Imag	Total
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

Approximate the real zeros of

$$f(x) = x^6 - 2x^5 + 3x^4 - 10x^3 - 6x^2 - 8x - 8$$

Homework: Ch. 5.7 pg.383 #'s
4,6,10-14e,22,26,34,52,
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